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**LETTERS AND COMMENTS**

# Comment on 'Note on Dewan–Beran–Bell's spaceship problem'

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**Abstract**
We present two objections to Redžić conclusion that in the 'tough variant' of Bell's thread-between-spaceships problem (the ships' acceleration is constant) the stretch of the thread remains finite. First, we show that because of the existence of the horizon for the accelerated observer Redžić drops out an essential part of the thread's history. Second, we show that there is no simple relation between the distance between the spaceship and the physical (leading to strain) stretch of the thread. We also present the correct estimate for the stretch, which shows that the stretch increases infinitely.

## 1. Introduction

In a recent note [1], Redžić revisits the well-known problem of 'Bell's spaceships' [2]. In the case of a constant acceleration of spaceships during an infinite time interval (which he calls the 'tough variant'), he comes to the conclusion that the thread stretch remains finite, and consequently the strong enough thread does not break. In this comment we show this conclusion to be incorrect.

## 2. The formulation of the problem and Redžić result

Bell's spaceship problem is formulated as follows. Let two spaceships be at rest in some inertial frame, with the distance between them equal to $H$. The spaceships are tied with a thread of length $H$, which is also at rest. At the moment $t = 0$, the spaceships simultaneously begin to move with an identical constant proper acceleration $a$. The question is: what happens to the thread? In particular, if it can withstand only a finite stretching, would it remain unbroken during an infinite period of acceleration or will it break sooner or later?

We can always adjust the spacetime scale so that the speed of light and the spaceships' acceleration are units, and the only remaining parameter is a dimensionless initial thread length

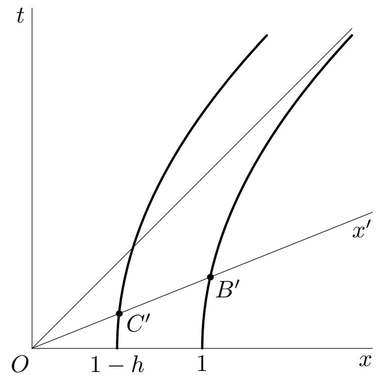


**Figure 1.** Minkowski diagram for Bell's problem.

$h = Ha/c^2$. Redžić in his paper writes down the dependence of the thread length $\Delta'_B$ on proper time $\tau_B$ of leading spaceship $B$ as

$$\Delta'_B = 1 + h \cosh \tau_B - \sqrt{1 + h^2 \sinh^2 \tau_B}, \tag{1}$$

and concludes that the relative stretch $\varepsilon_B = \Delta'_B / h$ has a finite limit,

$$\lim_{\tau_B \to \infty} \varepsilon_B = 1/h. \tag{2}$$

In what follows, we show that this conclusion is wrong on two accounts: first, because of the presence of a horizon for the accelerated observer Redžić's parametrization does not cover the whole motion history of the thread, and second, the value of $\Delta'_B$ itself cannot be used to determine the thread stretch. We also present a correct stretch estimate for the 'tough variant', which shows that the thread breaks sooner or later.

## 3. Horizon

As is well known, the world line of a body moving with constant proper acceleration is a hyperbole. In particular, the world line of spaceship $B$ in the inertial frame in which it was initially at rest at the point $x = 1$ (the 'resting frame') is described by the equation (see figure 1)

$$x_B^2 - t^2 = 1. \tag{3}$$

The world line of spaceship $C$ differs by a spatial shift,

$$(x_C + h)^2 - t^2 = 1. \tag{4}$$

The key point of Redžić's calculation is the introduction of another inertial frame, comoving with spaceship $B$ at the moment when its proper time equals $\tau_B$. It is well known that, if the spaceship is at this moment situated at $B'$, the $x'$-axis of the comoving frame passes through the origin $O$ of the resting frame and point $B'$. Redžić treats this line as a space slice of the spacetime, in particular, he takes the distance between the points $B'$ and $C$ along this axis as the thread length. However, it is easy to see that such a parametrization (proper time $\tau_B$ and distance along $x'$) covers only one half of the spacetime, namely, only the points that are separated from the resting frame origin $O$ by a spacelike interval. An essential part of the thread's history, lying above $x = t$, is completely dropped out; according to Redžić spaceship $C$ would *never* cross the line $x = t$ (there is no corresponding $\tau_B$ value). This is the true reason for the 'limiting' length of the thread; the distance $OB'$ always remains equal to unit, while the distance $OC'$ at large $\tau_B$ becomes negligible.

The above argument is reinforced by using the proper time $\tau_C$ of spaceship $C$ instead of that of spaceship $B$. Then

$$\Delta'_C = h \cosh \tau_C - 1 + \sqrt{1 + h^2 \sinh^2 \tau_C}, \tag{5}$$

which exhibits an infinite increase of the thread's length with proper time because now the parametrization covers the whole history of the thread.

## 4. Local stretch

The second objection to Redžić's note (as a matter of fact—to most of the papers on the subject) is that the quantity $\Delta'$ has nothing to do with the real thread stretch. The correct condition for a physical (leading to strain) deformation was formulated by Born in the early days of relativity: one has to look at the stretch in the comoving frame. On first sight that is what Redžić does. But the key feature of Bell's problem is that there exists no comoving frame that is common to both spaceships (except for the moment $t = 0$). In any inertial frame at any time at least one of the spaceships has non-zero velocity. Consequently there exists no comoving frame common to all points of the thread—every point requires its own comoving frame. Finally, there is no simple relation between the distance between the spaceships in some inertial frame and the stretch of the thread. Strictly speaking, the stretch 'for the whole thread' does not exist. The only meaningful quantity is a *local* stretch, which depends not only on time, but also on the spatial coordinate, $\varepsilon = \varepsilon(x, t)$. It is however possible to show that for any $t > 1 + h$ there exists a spacetime point, $(x^*, t^*)$, $0 < t^* < t$, lying between world lines of spaceships, such that

$$\varepsilon(x^*, t^*) > \sqrt{\frac{t}{2(1+h)}}. \tag{6}$$

Thus, the stretch increases infinitely at least at one point of the thread and so, sooner or later, the thread breaks.

Our conclusion is confirmed by a complete solution of the problem in a particular model of the thread. The proof of the estimate (6) and solutions of some other problems of the relativistic thread will be given in a more detailed paper.

## Acknowledgment

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## References

[1] Redžić D V 2008 Note on Dewan-Beran-Bell's spaceship problem *Eur. J. Phys.* **29** N11–N19
[2] Bell J S 1987 How to teach special relativity *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press) pp 67–80